$$\text{Power applied} = \frac{Fs}{t} = \frac{941 \times 300}{24} = 11{,}762 \text{ ft. lbs. per sec.} = 21 \text{ H.P. (approx.).}$$

283. Proposed by C. N. SCHMALL, New York City.

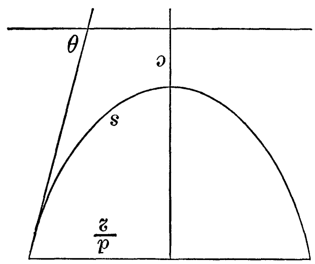
The maximum length of a certain chain which can be suspended from one end without breaking is l. It is desired to form a catenary with a length $2l/k$ of the chain, the points of support being a distance d apart, in the same horizontal line. Show that the maximum value of d is

$$\frac{2l}{k}(k^2-1)^{1/2}\log\left(\frac{k+1}{k-1}\right)^{1/2}.$$

SOLUTION BY A. M. HARDING, University of Arkansas.

Let w = weight per unit length. Then wl = maximum tension the chain will stand.

The tension at the point of support is given by $T\sin\theta = ws$ where s = one-half the length of the chain and θ is the angle that the tangent to the catenary at that point makes with the x-axis.



If we put $T = wl$ and $s = l/k$ we find $\sin\theta = 1/k$. But $\tan\theta = \frac{1}{2}(e^{d/2c} - e^{-(d/2c)})$ where c is the distance along the Y-axis from the origin to the catenary.

Hence we have

$$\tfrac{1}{2}(e^{d/2c} - e^{-(d/2c)}) = \frac{1}{\sqrt{k^2-1}}.$$

From this equation we obtain

$$d = 2c\log\left(\frac{k+1}{k-1}\right)^{\frac{1}{2}}.$$

We have also the intrinsic equation of the catenary $s = c\tan\theta$, from which we obtain

$$c = \frac{l}{k}\sqrt{k^2-1}.$$

Whence

$$d = \frac{2l}{k}(k^2-1)^{\frac{1}{2}}\log\left(\frac{k+1}{k-1}\right)^{\frac{1}{2}}.$$

Also solved by J. W. COLSON.

NUMBER THEORY.

189. Proposed by V. M. SPUNAR, Chicago, Illinois.

If p and $p_1 = 2^p - 1$ are primes, then are the numbers of the sequence $p_1 = 2^p - 1$, $p_2 = 2^{p_1} - 1$, $p_3 = 2^{p_2} - 1$, $\cdots$, $p_n = 2^{p_{n-1}} - 1$ all primes?

REMARKS BY R. D. CARMICHAEL, Indiana University.

If the conjecture stated in this problem is true it is highly desirable to have a proof of it. It would be a significant contribution to the theory of prime numbers; for the theorem so obtained would afford a means of recursion by which a sequence of indefinitely increasing prime numbers could be written out consecutively. No such tool has yet been invented. Euler and Legendre sought in vain for analytical expressions which would serve just this purpose. Fermat believed, though he confessed that he was unable to prove, that he had found such an analytical expression in

$$2^{2n} + 1.$$

Euler pointed out the error of this opinion by showing that 641 is a factor of this number for the case $n = 5$.

From these historical facts one is led to suppose that the proof of the conjecture of this problem is probably difficult, in case the conjecture is true; if the conjecture is false, that would be shown by means of an example. The numbers to be dealt with, however, are so large that the construction of such an example (in case it exists) would perhaps be very tedious.

Note. A number of incorrect solutions of problem 198 have been received. Will our contributors please give it a more careful consideration? It is very probably not easy to solve.

EDITORS.

199. Proposed by R. P. LOCHNER, Philadelphia, Pa.

Find three integral squares, such that the sum of every two of them shall be a square.—Alsop's Algebra, 3d edition (Philadelphia, 1859), p. 296, Ex. 13.

SOLUTION BY ARTEMAS MARTIN, Washington, D. C.

Let x^2, y^2 and z^2 be the required squares; then we must have

$$x^2 + y^2 = \square,\ x^2 + z^2 = \square,\ y^2 + z^2 = \square, \tag{1, 2, 3}$$

Assume $y = \dfrac{(m^2 - n^2)x}{2mn}$, $z = \dfrac{(p^2 - n^2)x}{2pn}$; then (1) and (2) are satisfied and (3) becomes, after striking out common square factors,

$$p^2(m^2 - n^2)^2 + m^2(p^2 - n^2)^2 = \square, \tag{4}$$

which may be otherwise written

$$m^2p^2(m^2 + p^2) - 4m^2n^2p^2 + n^4(m^2 + p^2) = \square; \tag{5}$$

and (5) will be a square when

$$m^2 + p^2 = 4n^2. \tag{6}$$

Put $m = \dfrac{2n(r^2 - s^2)}{r^2 + s^2}$, $p = \dfrac{4nrs}{r^2 + s^2}$; then (6) is satisfied. Take $r = 2$, $s = 1$, and we have $m = \dfrac{6n}{5}$, $p = \dfrac{8n}{5}$; $y = \dfrac{11x}{60}$, $z = \dfrac{39x}{80}$. Now take $x = 240$ and we have $y = 44$, $z = 117$, the least numbers known.